**Critical domain**

Our model can be used to predict the minimum diameter a mucus plug must be to allow for anaerobic bacteria to survive in its interior. The critical domain size problem asks how large a habitat must be to support a population, or the size a refuge must be for an animal to survive (13-15). Here, we consider how large a mucus plug must be to have a hypoxic region in which anaerobic bacteria can survive.

Assume oxygen is fixed in space and is distributed according to with corresponding to the air-mucus boundary. Consider the anaerobic population governed by

on the domain . The critical domain size is the minimum length of the domain such that *f* will go extinct if and *f* will have a non-trivial steady state if At steady state, , so the distribution of *f* is the solution of the ODE

where Since we can write , and letting the steady state distribution is the solution of the system

The Jacobian of this system is

with constant solutions at and . At both of these solutions we have

with eigenvalues .